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Minimal clusters in the plane with double densities

Abstract: The aim of this seminar is to present some results about the isoperimetric problem for clusters in the plane with double density. This amounts to finding the best configuration of \( m \) regions in the plane enclosing given volumes, in order to minimize their total perimeter, in the case where volume and perimeter are weighted by suitable densities. We focus on the so-called “Steiner” property, ensuring that boundaries of minimal clusters are made of regular curves meeting in triple points. In the standard Euclidean case the directions at triple points are at 120 degrees. We show that the Steiner property can be generalized to a wide class of densities under natural assumptions. If the perimeter density is isotropic, i.e., it does not depend on the normal, we show that triple points enjoy the usual 120 degrees Steiner property. For anisotropic densities the situation is more delicate and we discuss what possible directions occur in minimizers. Examples will be also discussed. This is a joint collaboration with A. Pratelli and G. Stefani.