Abstract: Classical results of Alexandrov (1958) and Korevaar-Ros (1988) characterize the ball as the unique smooth domain whose $k$-th mean curvature is constant for some $k = 1, \ldots, n$. Replacing the classical pointwise mean curvature functions by the curvature measures, the same uniqueness result is true among arbitrary convex bodies (Schneider 1979). In this talk we present a result that extends these theorems to the general class of sets of positive reach. Related sharp results on the removability of singularities are also presented.